UCRL-ID-119127

Implementation of a Rayleigh Scattering Optical Depth Parameterization

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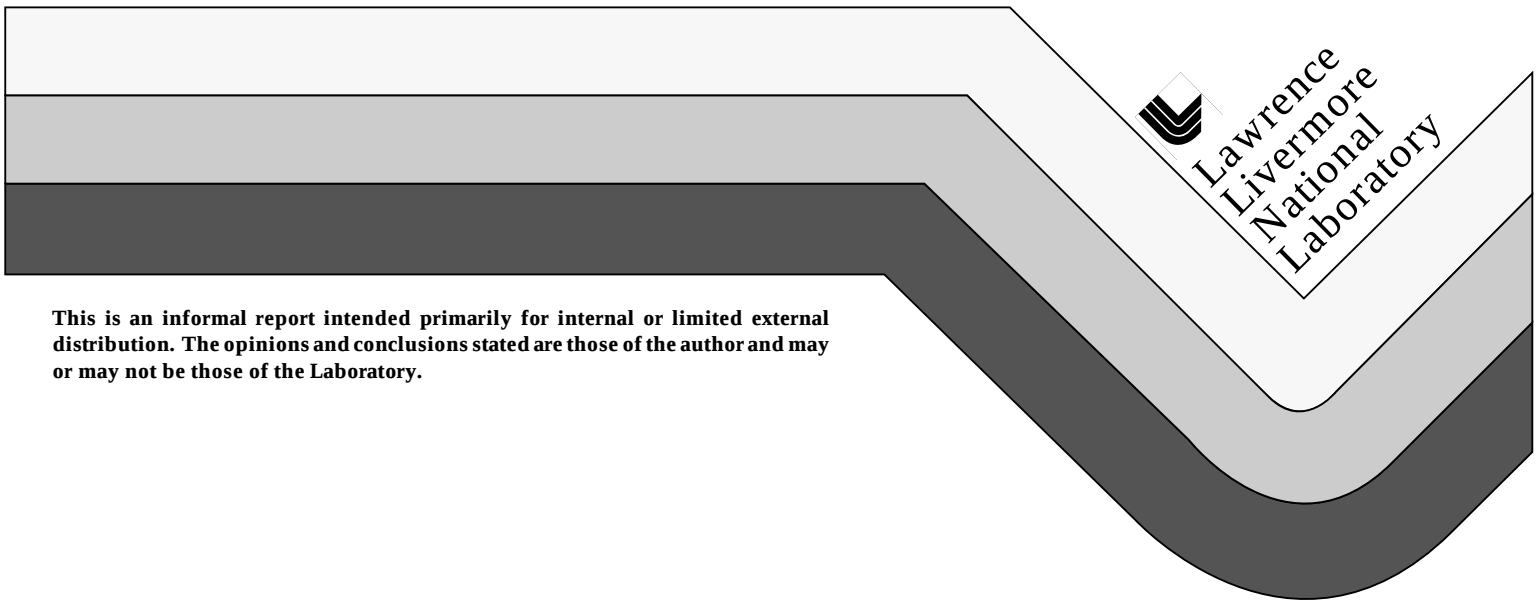
October 1994



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Implementation of a Rayleigh Scattering Optical Depth Parameterization

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A new procedure for calculating Rayleigh scattering cross sections and optical depths has been developed for use in the solar radiation transfer model of the LLNL Chemistry-Radiation-Transport Models. This procedure replaces the previous method for determining the optical depths in which the parameterization of Hansen and Travis (1974) was used to generate a table of optical depths for wavelengths ranging from 0.135 μm to 0.729 μm; the table was then placed directly in the shortwave package. Hansen and Travis calculate Rayleigh scattering optical depths, τ, using

$$\tau = 0.08569\lambda^{-4}\ (1 + 0.0113\lambda^{-2} + 0.00013\lambda^{-4}) \qquad \textbf{(1)}$$

where λ is wavelength in μm and the standard surface pressure is 1013 mB.

The new scheme uses instead the parameterization of Nicolet (1984) to find the Rayleigh scattering cross sections for an arbitrary set of wavelength bins, and from these cross sections, determines the optical depths. The Nicolet parameterization was recommended by the WMO (1986), and calculates Rayleigh scattering cross sections using

$$\sigma_{RS} = \frac{4.02x10^{-28}}{\lambda^{-(4\ x)}} \qquad \textbf{(2)}$$

where $x=0.389\lambda+0.09426/\lambda-0.3328$ for $\lambda<0.55$ μm and $x=0.04$ for $\lambda>0.55$ μm.

The average Rayleigh scattering cross section for a wavelength bin extending from λ_1 to λ_2 is found by integrating (2) across the bin

$$\langle\sigma_{RS}\rangle = (\lambda_2-\lambda_1)^{-1} \int_{\lambda_1}^{\lambda_2} \frac{4.02x10^{-28}}{\lambda^{-(4\ x)}}\ d\lambda\ . \qquad \textbf{(3)}$$

Since there is no obvious closed form for the solution to (3), an eight point Gaussian quadrature was used to evaluate the integral. From the cross sections obtained in this manner, the corresponding Rayleigh scattering optical depths to the bottom of the atmosphere, τ_{RB}, were calculated using

$$\langle\tau_{RB}\rangle = \langle\sigma_{RS}\rangle \int_z^\infty n(z)\ dz\ . \qquad \textbf{(4)}$$

$\langle\tau_{RB}\rangle$ is the average of τ_{RB} across the wavelength bin, and n(z) is the total number of absorbing molecules in a column of height z. n(z) is found by

$$\int_0^\infty n(z)\,dz = -\int_{P_0}^0 \left(\frac{N_0}{M_A\ g}\right) dP \approx \frac{N_0 P_0}{M_A g} \qquad \textbf{(5)}$$

where N_0 is Avogadro's number, M_A is the molecular weight of air, g is the acceleration of gravity, and P_0 is the surface pressure.

The accuracy of the Nicolet scheme was tested by comparing the Nicolet optical depths for a data set containing 126 new wavelength bins to the optical depths produced by the Hansen and Travis parameterization for those same bins. The optical depth data table from an earlier LLNL CRT Model consisted of 148 bins and their corresponding optical depths and was generated from (1). An interpolation was performed which mapped these 148 old wavelength bins and τ_{RB} onto the 126 new bins. Then, Rayleigh scattering cross sections and τ_{RB} were calculated for the new bins using the Nicolet parameterization. A comparison of the two sets of optical depths for the new wavelength bins is shown in figure 1.

The two sets of τ_{RB} agree well; the percentage difference between the two curves never grows larger than 10%. (See figure 2). The region of greatest deviation occurs at 0.2-0.4 μm, where τ_{RB} changes most rapidly.

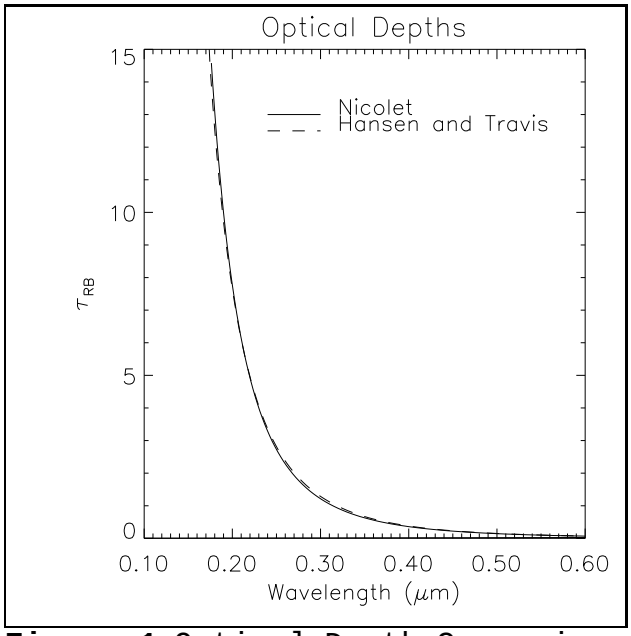

Figure 1 Optical Depth Comparison

The Nicolet optical depths were then compared to optical depths produced by using higher order Gaussian quadratures to solve (3). A quadrature of order 16 was used as a baseline to compare with quadratures of lower orders; it was assumed that the integral would have converged for quadrature orders lower than 16.

For the one point quadrature, the largest value of the percent difference from the 16 point quadrature is -0.25%. (See figure 3). For the two point cases the value of the maximum difference drops to -1.7×10^{-6} %, and for the four and eight point cases, the maximum differences are 5.5×10^{-8} % and 1.9×10^{-8} %, respectively. It is reasonable to assume that the eight point quadrature is sufficiently accurate for use in evaluating (3), as the values for the quadratures have converged by the time we reach the 8 point case.

The shape of the percent difference curves is determined in part by the structure of the wavelength bins. Bin spacing increases from 1.55×10^{-3} μm at 0.176 μm to 4.95×10^{-3} μm at 0.303 μm, and has a constant bin spacing of 5.0×10^{-3} μm for $\lambda > 0.303$ μm. In all cases, the percent difference between a given quadrature order and order 16 increases as the bin spacing increases,and then declines as the bin spacing becomes constant for $\lambda > 0.303$ μm. The peaks in the percent difference curves near 0.303 μm are caused by the rapid change of τ_{RB} in this region; the approximation to the integral has its lowest accuracy here.

The one point quadrature consistently underestimates the value

of τ_{RB}, and, as might be expected of a linear approximation, errs most where the τ_{RB} curve deviates most strongly from linearity. The two point case shows a similar behavior. The four and eight point quadratures overestimate the values of τ_{RB}, but show the same shape curve as the one and two point cases.

In conclusion, the Nicolet parameterization successfully calculates τ_{RB} for an arbitrary set of wavelength bins when the integral (3) is evaluated using an eight point Gaussian quadrature.
The Nicolet τ_{RB} agree well with those of Hansen and Travis for the case tested here.

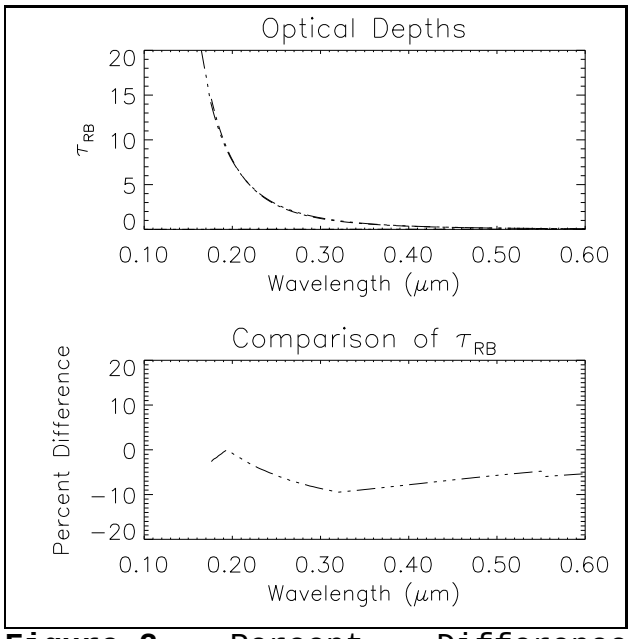

Figure 2 Percent Difference Curves

Acknowledgement

Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under contract No. W-7405-Eng-48 and was supported in part by both the Department o Energy Office of Environmental Analysis, the U.S. Environmental Protection Agency, and the Carbon Dioxide Research Program.

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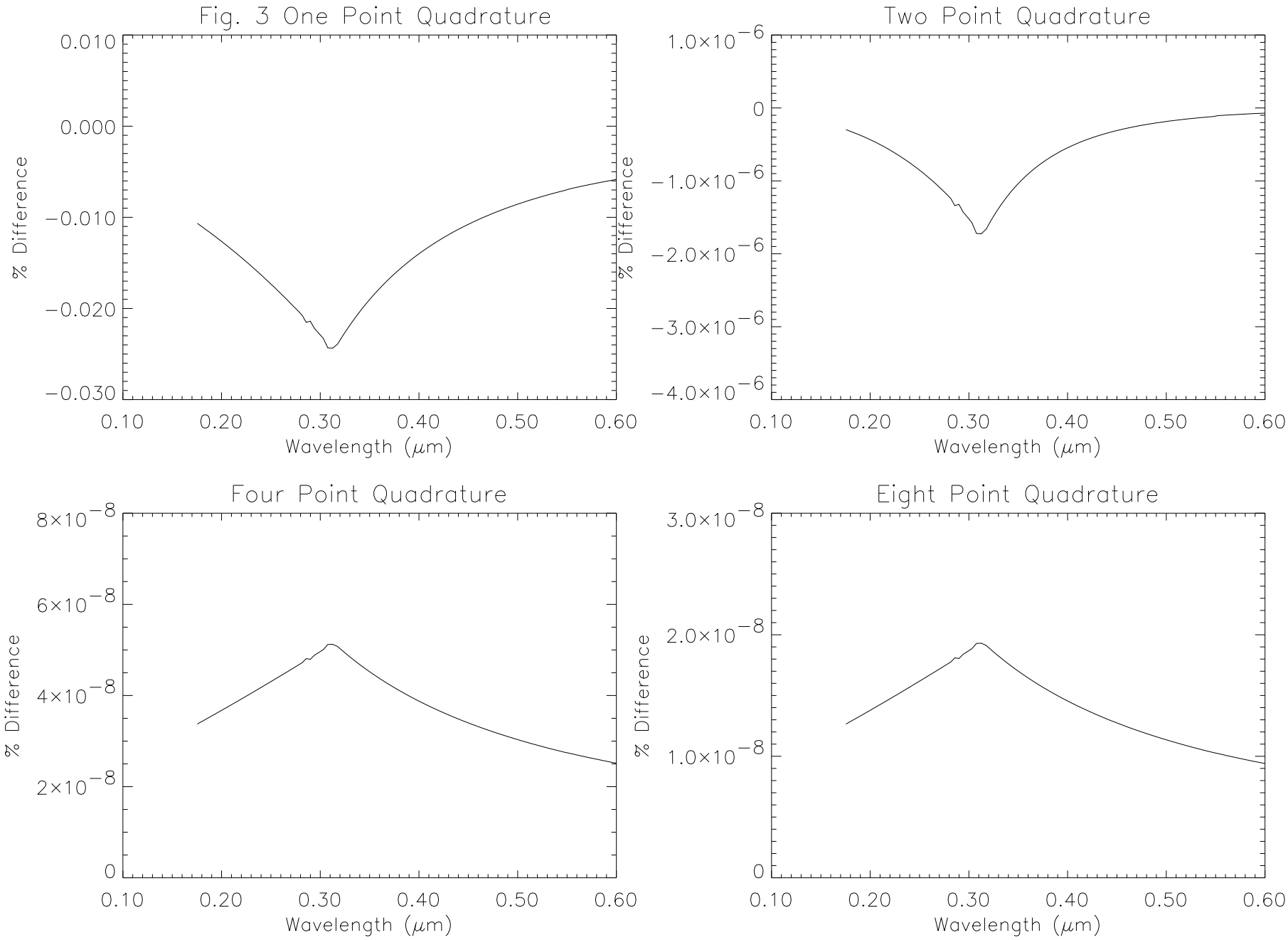
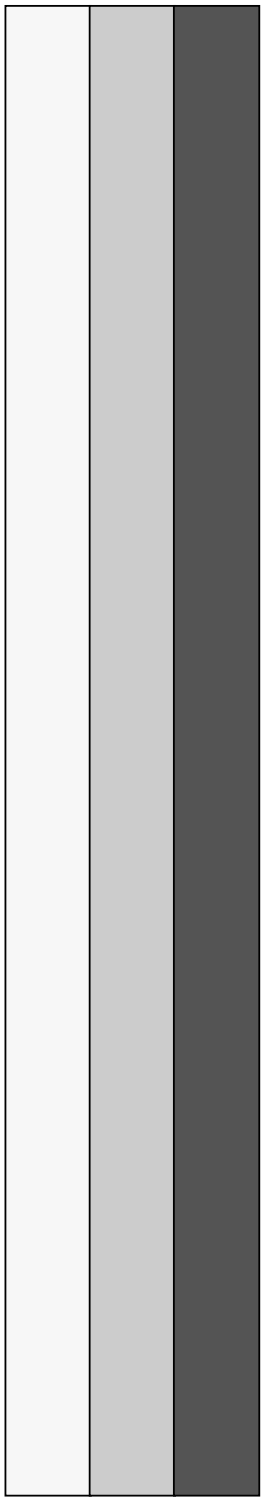


Fig. 3 One Point Quadrature
0.010
0.000
−0.010
−0.020
−0.030
% Difference
0.10
0.20
0.30
0.40
0.50
0.60
Wavelength (μm)
Two Point Quadrature
1.0×10−6
0
−1.0×10−6
−2.0×10−6
−3.0×10−6
−4.0×10−6
% Difference
0.10
0.20
0.30
0.40
0.50
0.60
Wavelength (μm)
Four Point Quadrature
8×10−8
6×10−8
4×10−8
2×10−8
0
% Difference
0.10
0.20
0.30
0.40
0.50
0.60
Wavelength (μm)
Eight Point Quadrature
3.0×10−8
2.0×10−8
1.0×10−8
0
% Difference
0.10
0.20
0.30
0.40
0.50
0.60
Wavelength (μm)

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